PHYSICS OF THE YO-YO

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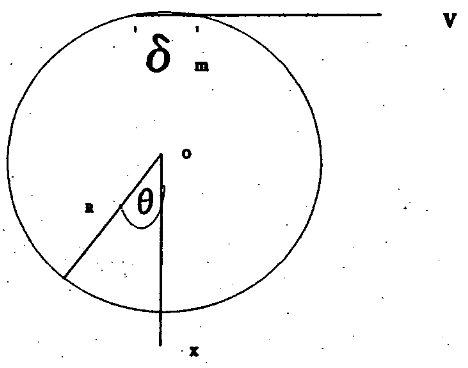
Yo-Yo enthusiasm is spreading in the country like a chain reaction. Yo-Yo is not only a handy toy, it beautifully illustrates some principles of dynamics. To understand physics of the Yo-Yo following prerequisites are needed.

Kinetic Energy of a Rotating Rigid Body

Everyone knows that kinetic energy of a particle moving with velocity V is $\frac{1}{2}mV^2$.

$$T = \tfrac{1}{2}mV^2$$

What is the kinetic energy of a circular hoop rotating with a velocity V.



Hoop may be regarded as made up of small elements of mass δm. The kinetic energy of one such element is $\frac{1}{2}\delta m\, V^2$. Thus the total kinetic energy is

$$T = \sum \tfrac{1}{2}\, \delta m\, V^2$$

$$= \tfrac{1}{2}V^2 \sum \delta m$$

$$= \tfrac{1}{2}MV^2 \qquad (1)$$

where M is the total mass of the hoop. A point on the rim of the hoop rotates through an angle 2π (360^0) in a time $2\pi R/V$. If OX is a line fixed in space, the rate of which the angle θ, varies with time is known as angular velocity, usually denoted by the symbol Ω. Clearly angular velocity of the hoop is

$$\Omega = 2\pi \div 2\pi R/V = \frac{V}{R} \qquad (1)$$

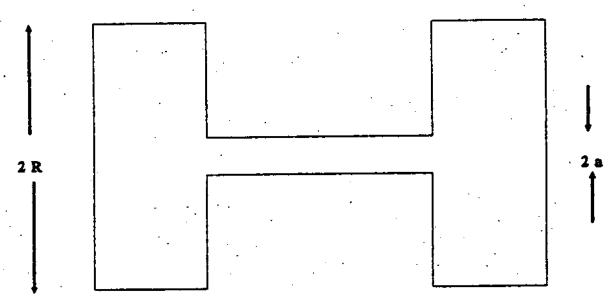
Therefore equation (1) can also be written in the form.

$$T = 1/2\,(MR^2)\,\Omega^2 \qquad (3)$$

The kinetic energy of a rotating disc can be worked out by dividing it into number of concentric rings. The result turns out to be :

$$T = 1/2 \quad (1/2MR^2)\,\Omega^2 \qquad (4)$$

Kinetic energy of a cylinder rotating about its axis is also given the formula (4).

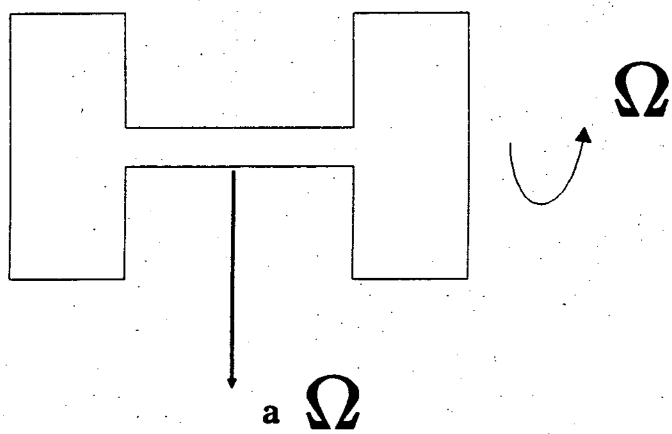


Neglecting mass and motion of the axis, rotational kinetic energy of Yo-Yo is also given by (4), where M is mass of the Yo-Yo.

Rotational and Translational Motion of the Yo-Yo

When the Yo-Yo rotates through an angle $\delta\theta$ it descends vertically a distance $\delta x = a\,\delta\theta$. Thus if V is the downward translational velocity and Ω is angular velocity we have

$$V = a\Omega \qquad (5)$$

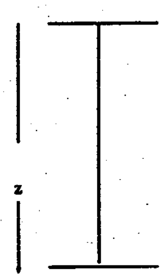


Therefore the total kinetic energy of the descending Yo-Yo is

$$\text{K.E (total)} = \text{K.E translation} + \text{K.E rotation}$$

$$T = T_T + T_R$$

ie, $T = \frac{1}{2}Ma^2\,\Omega^2 + \frac{1}{2}(\frac{1}{2}MR^2)\Omega^2$ (6)



When the Yo-Yo falls through a distance. z conservation of energy gives,

$$mgz = \frac{1}{2}\,Ma^2\Omega^2 + \frac{1}{4}MR^2\Omega^2 \qquad (7)$$

Equation (7) enables one to calculate the angular velocity of yo-yo when it has fallen a distance z.

If l is the length of the string, the maximum angular velocity attained by the Yo-Yo is Ω_l given by

(friction neglected)

$$Mgl = \tfrac{1}{2}Ma^2\,\Omega^2_l + \tfrac{1}{4}MR^2\,\Omega_l^2 \qquad (8)$$

As the string becomes taut, translational K.E is destroyed and string continues to wind up, Yo-Yo again acquires a upward velocity and moves up a distance d given by the equation.

$$Mgd = \tfrac{1}{4}MR^2\Omega_l^2 \qquad (9)$$

Clearly $d < l$.

When the Yo-Yo is projected downwards with a velocity u, the maximum angular velocity acquired is given by.

$$\tfrac{1}{2}Mu^2 + mgl$$
$$= \tfrac{1}{2}Ma^2\Omega^2_{lu} + \tfrac{1}{4}MR^2\Omega_{lu}^2 \qquad (10)$$

and distance it could rise up again is D, where

$$MgD = \tfrac{1}{2}MR^2\Omega^2_{lu} \qquad (11)$$

Depending on the value of u. D can exceed l. This explains the vertical up-down motion of the yo-yo. Kinetic energy of translation is destroyed at moment string becomes tant. However angular momentum about the axis of rotation is conserved. A yo-yo performs better when K.E of rotation is much greater than K.E of translation. The condition this is $R >> a$.

Other types of movements of the yo-yo can also explained, but requires a knowledge of more advanced dynamics. One example is rotation of the yo-yo with its plane horizontal. This is a result from an effect known as the 'gyroscopic action'. Rotating rigid body tends to maintain the axis of rotation in a fixed direction.