Markov processes, Hurst exponents, and nonlinear diffusion equations: With application to finance

Kevin E. Bassler[a], Gemunu H. Gunaratne[a,b], Joseph L. McCauley[a,c,*]

[a] *Physics Department, University of Houston, Houston, TX 77204, USA*
[b] *Institute of Fundamental Studies, Kandy, Sri Lanka*
[c] *COBERA, Department of Economics, J.E. Cairnes Graduate School of Business and Public Policy, NUI Galway, Ireland*

Received 6 December 2005; received in revised form 25 January 2006
Available online 24 February 2006

Abstract

We show by explicit closed form calculations that a Hurst exponent $H \neq \frac{1}{2}$ does not necessarily imply long time correlations like those found in fractional Brownian motion (fBm). We construct a large set of scaling solutions of Fokker–Planck partial differential equations (pdes) where $H \neq \frac{1}{2}$. Thus Markov processes, which by construction have no long time correlations, can have $H \neq \frac{1}{2}$. If a Markov process scales with Hurst exponent $H \neq \frac{1}{2}$ then it simply means that the process has nonstationary increments. For the scaling solutions, we show how to reduce the calculation of the probability density to a single integration once the diffusion coefficient $D(x,t)$ is specified. As an example, we generate a class of student-t-like densities from the class of quadratic diffusion coefficients. Notably, the Tsallis density is one member of that large class. The Tsallis density is usually thought to result from a nonlinear diffusion equation, but instead we explicitly show that it follows from a Markov process generated by a linear Fokker–Planck equation, and therefore from a corresponding Langevin equation. Having a Tsallis density with $H \neq \frac{1}{2}$ therefore does not imply dynamics with correlated signals, e.g., like those of fBm. A short review of the requirements for fBm is given for clarity, and we explain why the usual simple argument that $H \neq \frac{1}{2}$ implies correlations fails for Markov processes with scaling solutions. Finally, we discuss the question of scaling of the full Green function $g(x,t;x',t')$ of the Fokker–Planck pdes.
© 2006 Elsevier B.V. All rights reserved.

Keywords: Hurst exponent; Markov process; Scaling; Stochastic calculus; Autocorrelations; Fractional Brownian motion; Tsallis model; Nonlinear diffusion