Hurst exponents, Markov processes, and fractional Brownian motion

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Abstract

There is much confusion in the literature over Hurst exponents. Recently, we took a step in the direction of eliminating some of the confusion. One purpose of this paper is to illustrate the difference between fractional Brownian motion (fBm) on the one hand and Gaussian Markov processes where $H \neq \frac{1}{2}$ on the other. The difference lies in the increments, which are stationary and correlated in one case and nonstationary and uncorrelated in the other. The two- and one-point densities of fBm are constructed explicitly. The two-point density does not scale. The one-point density for a semi-infinite time interval is identical to that for a scaling Gaussian Markov process with $H \neq \frac{1}{2}$ over a finite time interval. We conclude that both Hurst exponents and one-point densities are inadequate for deducing the underlying dynamics from empirical data. We apply these conclusions in the end to make a focused statement about 'nonlinear diffusion'.

Keywords: Markov processes; Fractional Brownian motion; Scaling; Hurst exponents; Stationary and nonstationary increments; Autocorrelations