

Is integration $I(d)$ applicable to observed economics and finance time series?

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ABSTRACT

The method of cointegration in regression analysis is based on an assumption of stationary increments. Stationary increments with fixed time lag are called 'integration $I(d)$ '. A class of regression models where cointegration works was identified by Granger and yields the ergodic behavior required for equilibrium expectations in standard economics. Detrended finance market returns are martingales, and martingales do not satisfy regression equations. We ask if there exist detrended processes beyond standard regression models that satisfy integration $I(d)$. We show that stationary increment martingales are confined to the Wiener process, and observe that martingales describing finance data admit neither the integration $I(d)$ nor the ergodicity required for long time equilibrium relationships. *In particular, the martingales derived from finance data do not admit the time (or 'space') translational invariance required for increment stationarity.* Our analysis explains the lack of equilibrium observed earlier between FX rates and relative price levels.