

# PHYSICS OF THE YO-YO

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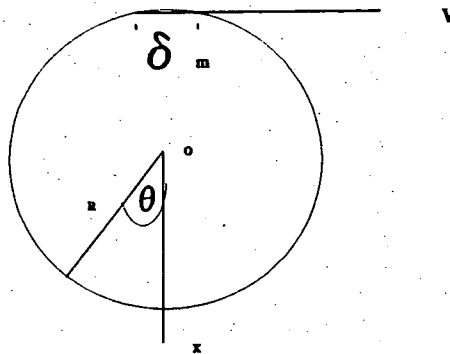
Yo-Yo enthusiasm is spreading in the country like a chain reaction. Yo-Yo is not only a handy toy, it beautifully illustrates some principles of dynamics. To understand physics of the Yo-Yo following prerequisites are needed.

## Kinetic Energy of a Rotating Rigid Body

Everyone knows that kinetic energy of a particle moving with velocity  $V$  is  $\frac{1}{2}mV^2$ .

$$\text{---} \bullet \text{---} \rightarrow \quad T = \frac{1}{2}mV^2$$

What is the kinetic energy of a circular hoop rotating with a velocity  $V$ .



Hoop may be regarded as made up of small elements of mass  $\delta m$ . The kinetic energy of one such element is  $\frac{1}{2}\delta m V^2$ . Thus the total kinetic energy is

$$\begin{aligned} T &= \sum \frac{1}{2} \delta m V^2 \\ &= \frac{1}{2} V^2 \sum \delta m \\ &= \frac{1}{2} M V^2 \end{aligned} \quad (1)$$

where  $M$  is the total mass of the hoop. A point on the rim of the hoop rotates through an angle  $2\pi$  ( $360^\circ$ ) in a time  $2\pi R/V$ . If  $OX$  is a line fixed in space, the rate of which the angle  $\theta$ , varies with time is known as angular velocity, usually denoted by the symbol  $\Omega$ . Clearly angular velocity of the hoop is

$$\Omega = 2\pi \div 2\pi R/V = \frac{V}{R} \quad (1)$$

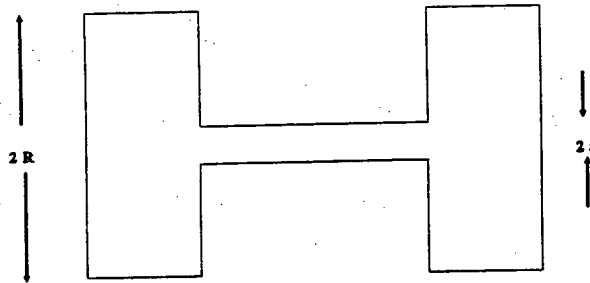
Therefore equation (1) can also be written in the form.

$$T = 1/2 (MR^2) \Omega^2 \quad (3)$$

The kinetic energy of a rotating disc can be worked out by dividing it into number of concentric rings. The result turns out to be :

$$T = 1/2 (1/2MR^2) \Omega^2 \quad (4)$$

Kinetic energy of a cylinder rotating about its axis is also given the formula (4).

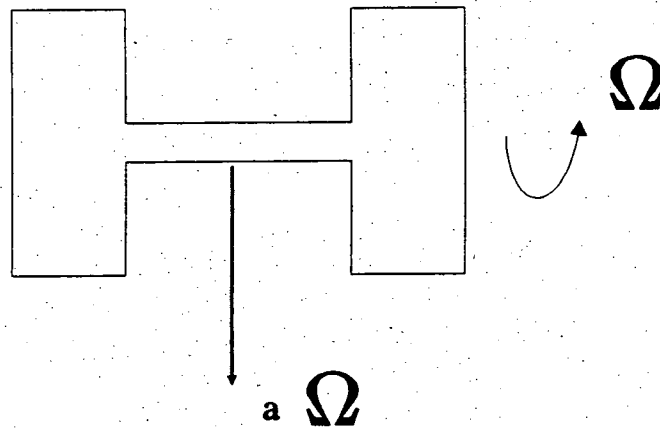


Neglecting mass and motion of the axis, rotational kinetic energy of Yo-Yo is also given by (4), where M is mass of the Yo-Yo.

### Rotational and Translational Motion of the Yo-Yo

When the Yo-Yo rotates through an angle  $\delta\theta$  it descends vertically a distance  $\delta x = a \delta\theta$ . Thus if V is the downward translational velocity and  $\Omega$  is angular velocity we have

$$V = a\Omega \quad (5)$$

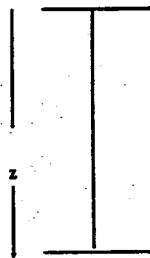


Therefore the total kinetic energy of the descending Yo-Yo is

$$\text{K.E (total)} = \text{K.E translation} + \text{K.E rotation}$$

$$T = T_T + T_R$$

$$\text{ie, } T = \frac{1}{2}Ma^2 \Omega^2 + \frac{1}{2}(\frac{1}{2}MR^2)\Omega^2 \quad (6)$$



When the Yo-Yo falls through a distance.  $z$  conservation of energy gives,

$$mgz = \frac{1}{2} Ma^2\Omega^2 + \frac{1}{4}MR^2\Omega^2 \quad (7)$$

Equation (7) enables one to calculate the angular velocity of yo-yo when it has fallen a distance  $z$ .

If  $l$  is the length of the string, the maximum angular velocity attained by the Yo-Yo is  $\Omega_1$  given by

(friction neglected)

$$Mgl = \frac{1}{2}Ma^2 \Omega_1^2 + \frac{1}{4}MR^2 \Omega_1^2 \quad (8)$$

As the string becomes taut, translational K.E is destroyed and string continues to wind up, Yo-Yo again acquires a upward velocity and moves up a distance  $d$  given by the equation.

$$Mgd = \frac{1}{4}MR^2 \Omega_1^2 \quad (9)$$

Clearly  $d < l$ .

When the Yo-Yo is projected downwards with a velocity  $u$ , the maximum angular velocity acquired is given by.

$$\begin{aligned} \frac{1}{2}Mu^2 + mgl \\ = \frac{1}{2}Ma^2 \Omega_{lu}^2 + \frac{1}{4}MR^2 \Omega_{lu}^2 \end{aligned} \quad (10)$$

and distance it could rise up again is  $D$ , where

$$MgD = \frac{1}{2}MR^2 \Omega_{lu}^2 \quad (11)$$

Depending on the value of  $u$ .  $D$  can exceed  $l$ . This explains the vertical up-down motion of the yo-yo. Kinetic energy of translation is destroyed at moment string becomes taut. However angular momentum about the axis of rotation is conserved. A yo-yo performs better when K.E of rotation is much greater than K.E of translation. The condition this is  $R \gg a$ .

Other types of movements of the yo-yo can also explained, but requires a knowledge of more advanced dynamics. One example is rotation of the yo-yo with its plane horizontal. This is a result from an effect known as the 'gyroscopic action'. Rotating rigid body tends to maintain the axis of rotation in a fixed direction.