FERMAT'S LAST THEOREM

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Everyone is familiar with Pythagoras Theorem named after the Greek philosopher and mathematician, Pythagoras of Samos, who is believed to be a contemporary of Lord Buddha. This theorem states that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the two adjacent sides. Pythagoras noted integral lengths of sides of right-angled triangles (e.g. adjacent sides 3,4 units and hypotenuse 5 units) satisfying the above condition. The general proof of the theorem was provided later by other Greek mathematicians.

A set of integers (x,y;z) which satisfies the equation $x^2 + y^2 = z^2$ are referred to as pythagorean numbers. (3,4,5) is one such example. If m and n are two integers, Pythagorean numbers can be expressed in the form :

$$x = m^2 - n^2 \qquad (1)$$

$$y = 2mn \qquad (2)$$

$$z = m^2 + n^2 \qquad (3)$$

It is easy to verify that with the above choice $x^2 + y^2 = z^2$. When m = 2, n = 1, one obtains the set (3,4,5). From the above it is clear that non-zero integral values x,y,z could be found to agree with the relation $x^2 + y^2 = z^2$. An intriguing question is whether the equation.

$$x^n + y^n = z^n \qquad (4)$$

could also be satisfied by non-trivial (ie, ones for which at x,y,z are all non-zero) integral values of x,y,z for $n > 2$. **The statement referred to as "Fermat's Last Theorem" is the assertion that for $n > 2$, non-trivial integral values x,y,z satisfying equation $x^n + y^n = z^n$ do not exist.**

Piere de Fermat (1601-1665) the French mathematician after whom the above theorem is named, was born into a family of leather merchants living near the city of Toulouse in France. He studied law and became an attorney in 1631. In his spare time he studied mathematics and made remarkable contributions to age old problems in mathematics. Fermat's favourite book was *"Arithmetica"* a latin translation of a Greek mathematics book by Diophantus. In a margin of the copy of *Arithmetica* used by Fermat, the above theorem was written with the following comment. I have found a marvelous proof of this theorem, but the narrow margin of the book is not sufficient to write out the proof. Since then, for three centuries mathematicians all over the world have burnt mid night oil to attack this problem, but with no success. The simplicity of the problem and the utmost difficulty of giving a proof attracted all those interested in mathematics. It is said that almost every day someone somewhere in the world claims to have proved Fermat's Last Theorem. Invariably, all these were shown to be groundless. The submission of proofs of Fermat's Last Theorem became such an headache to the French Academy of Sciences which decided sometime ago that it could no longer consider claims of the proofs of the Last Theorem.

Over three centuries other mathematics genii were born. Among them are Cauchy, Gauss, Weierstrass and Hilbert. They were either unsuccessful or indifferent to solution of the Last Theorem. Nearly one century after the death of Fermat, the great German mathematician Gauss was asked whether he had attempted to prove the Last Theorem. The reply was, he could propose hundreds of problems which neither be proved nor disproved. He went on further to say that paying attention to such problems impede the progress of mathematics. Despite the interest in Fermat's Last Theorem continued. The Theorem was proved for several special values of n. In the computer age empirical validity of the Theorem was established.

This age old mathematical puzzle was last resolved by Professor Andrew Wiles a British mathematician working in United States. The proof has been accepted by other mathematicians. However, minor improvements may be needed for perfection of the proof. The hope is that it will be completed in about year or two.

Contrary to the belief of Gauss, the proof of the Last Theorem is expected to have a profound impact on the progress of mathematics. Branches of mathematics known as algebraic geometry, number theory,. representation theory and theory of automorphic functions in the classical framework are independent domains. The outcome of the proof of Last Theorem may lead to conceptual unification of the above apparently disparate fields. A question that could arise is did Fermat actually prove the theorem. Most experts think that Fermat did not have a general proof. But this will not dwindle the greatness of Pieve de Fermat.

Nowadays people are more concerned with practical utility of new findings. Solving Fermat's Last Theorem has no immediate practical application. The same is true of other findings in mathematics as well as in physics, at the time of discovery. Intellectual and technological advancement of mankind cannot be separated, they are interlinked and more forward hand-in-hand.